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Fuzzy Entropy: An application to a model of fuzzy business diagnosis

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ABSTRACT

This paper extends the theory of fuzzy diseases´ predictions in order to improve cause detection by introducing Fuzzy Entropy, as an alternative measure for the analysis of causes. In this case, the fuzzy entropy allows to assess a level of uncertainty in the expert´s valuation of the firm and the visualization of certain problems in experts' valuation levels, which can cause higher levels of relative uncertainty in the analysis of causes in business diagnosis. Also, with this extension, the model can be useful to develop suitable computer systems for monitoring companies' problems, warning of failures and facilitating decision-making.

Keywords: Matrix of economic–financial knowledge; Economic–financial diagnosis; Symptoms and causes; Fuzzy relations; Fuzzy Entropy.

1 INTRODUCTION

The economic and financial diagnosis of companies, together with the prediction of insolvency, has been extensively discussed in the literature. The first models to predict the insolvency or failure of a company originated in the late sixties, from [1] and [2], and basically compare and classify companies according to quantitative indicators to distinguish between healthy and unhealthy businesses. The methodologies for constructing the rankings of healthy and unhealthy companies have been developed using a variety of models that combine different statistical and mathematical techniques (logit, probit, neural networks, expert systems, fuzzy sets, hybrid systems, genetic algorithms, clusters, survival models, data envelopment analysis, support vector machines, among others).

However, due to the nature of the diagnosis problem, typically subjective because it uses a large number of qualitative variables or expert analysts' opinions, it is very difficult to find a comprehensive solution using a classical method. That is why this field is suitable for tools taken from fuzzy logic, which allow working with qualitative variables, weak information and measuring the expert's knowledge. The fuzzy financial economic diagnostic models emerge as an alternative in this context to overcome many of the restrictions of traditional models [3].

The original fuzzy diagnostic model, through the use of fuzzy binary relations, was published in 2008 [4]. The model received a set of improvements that can be seen reflected in different contributions [5,6,7,8,9], and finally synthesized in [3]. The model developed, presents an analogy with the medical diagnosis of patients through the relationship between causes and symptoms [10], and contributions of that author are used, for the determination of the R matrix of financial economic knowledge. This work incorporates the notion of Fuzzy Entropy, as an alternative measure for the analysis of causes, which allows to assess a level of uncertainty in the valuations of firms by the experts.

2 FIRM ANALYSIS

Following the model of Sánchez [10], the main analytical goal is to determine matrix R, which in this case is a matrix of economic–financial knowledge represented as a fuzzy binary relation between symptoms and causes.

Definition 1 (Fuzzy binary relation). A fuzzy binary relation between two non-empty sets X and Y, is a fuzzy set R of X×Y, if: R : X×Y →[0,1]. Let F(X×Y) = {R : X×Y →[0,1]}.The value R (x,y) is interpreted as the degree or intensity of the relation R between x and y.

Therefore, the elements in matrix R represent the degree in which the occurrence of a symptom implies the occurrence, in a certain level, of a certain cause (disease). In the literature, the symptoms in economic–financial diagnosis can be represented as ratios, while the causes are the problems which generate the relative state of the symptoms. In an analogy to medicine, at the moment of diagnosis the causes are unknown, while the symptoms are known; thus, knowing the relative state of symptoms will serve to determine the relative state of causes. In this way, fuzzy models not only determine the good or bad health of a company, but also the reason why a company may be sick.

2.1. THE SYMPTOMS

The set of symptoms *S* consists of various symptoms, , i.e., $S=\{s_i\}$, where $i=1, 2,\ldots,n$. The symptoms are associated with economic and financial ratios. These ratios reflect aspects of profitability, productivity, liquidity, leverage, solvency, financial structure, debt coverage, economic structure, activity, turnover, efficiency and self-financing. Also, for companies listed on the capital markets, aspects related to shares and yields can be included. The ratios can be easily measured and taken from the firm's financial statements. A recommended list of symptoms is presented in an annex, at the end of the work or http://fuzzybusinessdiagnosis.blogspot.com.ar.

Using the ratios, the cardinal matrix of symptoms (*S*) is obtained that shows the nominal level of the symptoms for each company i.e., $E=\{E_h\}$, where $h=1,2,3,\ldots,m$.

$$S=\begin{array}{c|c|c|c|c|c|c|} & s_1 & s_2 & - & s_i & - & s_n \\ \hline E_1 & s_{11} & s_{12} & -- & s_{1i} & - & s_{1n} \\ \hline E_2 & s_{21} & s_{22} & -- & s_{2i} & - & s_{2n} \\ \hline --- & --- & --- & -- & --- & - & --- \\ \hline E_h & s_{h1} & s_{h2} & -- & s_{hi} & - & s_{hn} \\ \hline --- & --- & --- & -- & --- & - & --- \\ \hline E_m & s_{m1} & s_{m2} & -- & s_{mi} & - & s_{mn} \\ \hline \end{array} \tag{1}$$

Having the nominal set of symptoms, the following procedure is applied to obtain the membership function that reflects the incidence of each symptom over the firm.

(i) Determine the sign of property p_i with respect to symptom s_i. For example, if symptom s_i is liquidity, its sign is positive, so a higher liquidity represents a better financial condition for the firm. Therefore, the lowest membership level (that is, the firm shows a low intensity level of the symptom) corresponds to the most liquidity firm; in other words, the firm exhibits a low level of liquidity problems. According to this, the determination of the sign is extended to all the ratios of matrix *S*.

(ii) Then, establish a complete ordering on each symptom s_i according to the sign determined in (i). That is, if the sign of the property is positive, the elements are ordered from highest to lowest; and if the sign of the property is negative, the elements are ordered from lowest to highest.

(iii) Once the elements have been ordered, the incidence level of symptom s_i for firm E_h, s_{hi}, is estimated by the ratio between the ordinal—within the ordering established in (ii)—of symptom s_i for that firm and the cardinal of the set (i.e., the number of firms).

That is:

$$q_{hi} = |s_{hi}|/|E_h| = |s_{hi}|/m \tag{2}$$

Example 1. If the return on assets (ROA) of five firms are $s=\{s_1, s_2, s_3, s_4, s_5\}$
s= {0.11, 0.05, 0.31, 0.46, 0.18} for $\{E_1, E_2, E_3, E_4, E_5\}$.
They are ordered from highest to lowest (0.46> 0.31> 0.18> 0.11> 0.05)
Hence, the level of incidence of the symptom s_i is $q=\{q_1, q_2, q_3, q_4, q_5\}$
q= {0.20, 0.40, 0.60, 0.80, 1.00} for $\{E_4, E_3, E_5, E_1, E_2\}$.

(iv) After repeating this procedure for every symptom, the membership matrix of symptoms, $Q= [q_{hi}]$, is obtained. The order of this matrix is $m\times n$ (m firms by n symptoms) and it shows the intensity levels of each symptom for each firm.

$$Q = \begin{array}{c} \\ E_1 \\ E_2 \\ --- \\ E_h \\ --- \\ E_m \end{array} \begin{array}{cccccc} s_1 & s_2 & -- & s_i & - & s_n \\ \hline q_{11} & q_{12} & -- & q_{1i} & - & q_{1n} \\ q_{21} & q_{22} & --- & q_{2i} & - & q_{2n} \\ --- & --- & --- & --- & - & --- \\ q_{h1} & q_{h2} & --- & q_{hi} & - & q_{hn} \\ --- & --- & --- & --- & - & --- \\ q_{m1} & q_{m2} & --- & q_{mi} & - & q_{mn} \end{array} \qquad (3)$$

2.2. THE CAUSES

While symptoms are considered to be the precursor signals of a possible disaster that are shown in financial statements, causes are the factors that generate problems. C is defined as the set of causes, i.e., $C = \{c_j\}$, where $j =1,2,...,p$. These causes, objective and subjective measured, are detected through a mapping methodology using the Balanced Scorecard- BSC- following [6]. This allows detecting a vast list of causes classified according to the key areas stablished in the BSC. This use of the BSC allows a permanent and continuous monitoring of all the areas of the firm. And this instrument that is known and used by the firms facilitates the auto-diagnosis (i.e. The key areas for the tracking may be business learning, innovation and technology, labor quality, cost optimization, results of activity, risk management, use of assets, technical efficiency, purchase policies, logistics, exogenous change, commercial management, consumer satisfaction, quality and prices, and sectorial evolution). Consequently, there are multiple factors or causes within each key area that can be disaggregated and measured either objectively or subjectively.

Subjective causes are proposed to be obtained by constructing linguistic labels, in a scale between [0, 1] that reflect the opinion of experts about the impact of the cause over the firm's performance. According to [11] higher degrees correspond to causes that have greater incidence level.

That means:

•A set of linguistic labels is given to each expert, with which the existence of the cause in each firm must be assessed.

•The expert chooses between the groups of linguistic labels that are translated into a quantitative scale which shows the incidence level of the cause.

•Each label represents a level of incidence that depends on the number of alternatives, or linguistic labels making up the scale. This incidence level is constructed through the cumulative frequency of the label in each scale.

Example 2. Let's suppose there are five states sorted from lowest to highest (always, often, sometimes, rarely and never), the scale is distributed linearly among the five states, corresponding 20% of the scale to each state. The quantitative scale is {0.20; 0.40; 0.60; 0.80; 1.00}. If there are seven states, the incidence levels would be {0.14; 0.29; 0.43; 0.57; 0.71; 0.86; 1.00}. This means that the incidence level changes according to the number of states established by the researcher for each cause. Also, it is possible to use the same number of states for all causes.
The objective causes must be ordered in the sense of impact (if the sense is positive, they are ordered from lowest to highest, and if it is negative, they are ordered from the highest to the lowest level). Then, once the elements are ordered, the cause incidence level is estimated through the ratio between the ordinal cause c_j established for the firm in the order and the cardinal of the set, which is the number of firms.
That is:

$$p_{hj} = |c_{hj}| / m \qquad (4)$$

After repeating the procedure for every cause, the membership matrix of causes, $P= [p_{hj}]$, is obtained.

$$P= \begin{array}{c} \\ E_1 \\ E_2 \\ --- \\ E_h \end{array} \begin{array}{cccccc} c_1 & c_2 & -- & c_j & - & c_p \\ \hline p_{11} & p_{12} & - & p_{1j} & - & p_{1p} \\ p_{21} & p_{22} & -- & p_{2j} & - & p_{2p} \\ --- & --- & -- & --- & - & --- \\ p_{h1} & p_{h2} & -- & p_{hj} & - & p_{hp} \end{array} \qquad (5)$$

---	---	---	--	---	-	---
E_m	p_{m1}	p_{m2}	--	p_{mj}	-	p_{mp}

2.3. Economic-financial Knowledge Matrix R for One Period

Once matrices *Q* and *P* have been constructed, which measure, respectively, the incidence of symptoms and causes for each one of the *m* firms for a given year, the next step is to find matrix *R*. The elements of $R=\{r_{ij}\}$ measure the intensity of the incidence of the symptom s_i over the cause c_j.

R matrix is obtained through $P = Q \circ R$, where "$\circ$" is the composition of two fuzzy relations *Q* and *R*. Given that *P* and *Q* are known, therefore the method for solving the fuzzy equations P and Q are known, so the method for solving fuzzy equations developed by [10] is used to find the largest solution presented in (6).

$$R = Q^T \alpha P, \qquad (6)$$

Where,

$$Q^T = [q_{hi}]^T = [q_{ih}],$$

That is:

$$R = Q^T \alpha P = [q_{ih}] \alpha [p_{hj}] = [r_{ij}].$$

Where, following [10], the operation $R = Q^T \alpha P$ is defined as

$$[r_{ij}] = h [q_{ih} \alpha p_{hj}] \qquad (7)$$

Where,

If $q_{ih} < p_{hj}$ $\quad r_{ij} = q_{ih} \alpha p_{hj} = 1$

If $q_{ih} > p_{hj}$ $\quad r_{ij} = q_{ih} \alpha p_{hj} = p_{hj}$

Through this operation, the *R* matrix of economic-financial knowledge is calculated using the *α* fuzzy relations operator or Godel implication. Thus, the analysts use their knowledge and the information available (either historical or prospective) to determinate *R*, $R \in F(Q \times P)$, where *R* is a matrix of order $n \times p$ and represents the fuzzy relation between symptoms and causes. Each element of matrix $R(r_{ij})$ represents the degree (or intensity level) to which a symptom s_i implies a cause c_j; and it is represented by the value of the r_{ij}, where $r_{ij} \in [0, 1]$. Therefore, the matrix *R* can be represented as: $R = [r_{ij}]$ with $i = 1,...,n$ and $j = 1,...,p.$, according to (5).

$$R = \begin{array}{c|cccccc} & c_1 & c_2 & - & c_j & - & c_p \\ \hline s_1 & r_{11} & r_{12} & -- & r_{1j} & - & r_{1p} \\ s_2 & r_{21} & r_{22} & -- & r_{2j} & - & r_{2p} \\ -- & --- & --- & - & -- & - & --- \\ s_i & r_{i1} & r_{i2} & - & r_{ij} & - & r_{ip} \\ -- & --- & --- & - & -- & - & --- \\ s_n & r_{n1} & r_{n2} & - & r_{nj} & - & r_{np} \end{array} \qquad (8)$$

2.4. The Aggregate Matrix ($\Re$)

Having found R^*, that is, the matrix *R* for a given year, from which any possible inconsistencies have been removed, the next step is the aggregation of the matrices R^* computed for each period T^k. By this a matrix of economic-financial knowledge ($\Re$) is obtained that is representative of all firms and every year under consideration.

The choice of the best operator for the aggregation depends on whether the series of matrices for each year exhibits a trend. The behavior of each r_{ij} is evaluated in order to determine the aggregation process depending on the trend experienced by each component.

If $\Sigma|[r_{ij}^k - r_{ij}^{k-1}]| = 0$; r_{ij}^k = aggregate r_{ij} ($\check{r}_{ij}$)
If $\Sigma|[r_{ij}^k - r_{ij}^{k-1}]| \neq 0$; $\check{r}_{ij}$ is determined by the indicator ξ, which varies between –1 and 1.

Thus, before choosing an operator, it is necessary to analyze whether these trends exist according to Eq. 7.

$$\xi = \Sigma_{k=2}\left([r_{ij}]_k - [r_{ij}]_{k-1}\right) / \Sigma_{k=2}\left|[r_{ij}]_k - [r_{ij}]_{k-1}\right| \qquad (9)$$

Then,

If ξ= 1, an increasing trend is assumed and the operator (10) must be used for the aggregation.

$$\check{r}_{ij} = ((r_{ij})_{k-1} \circ (r_{ij})_k) = max(min\ ((r_{ij})_{k-1}\ ;\ (r_{ij})_k). \qquad (10)$$

If ξ= -1, a decreasing trend is observed and the operator (11) is proposed.

$$\check{r}_{ij} = ((r_{ij})_{k-1} \circ (r_{ij})_k) = \ min\ (max\ ((r_{ij})_{k-1}\ ;\ (r_{ij})_k)) \qquad (11)$$

Since there is an increasing or decreasing trend (and in order not to amplify) it is taken a conservative operator, the minimum or the maximum that reflects the trend (maximum of the minimum or the minimum of the maximum).

If -1< ξ< 1, there is no trend and the generalized means operator is proposed to be used for aggregation according to Eq. (12). That is because it is a continuous, symmetric and idempotent aggregation operator.

$$\check{r}_{ij} = \left[\sum_{k=1}^{t}\left(r_{ij}{}_k^{\varphi}\right)/k\right]^{1/\varphi} \qquad (12)$$

Having chosen the best operator for aggregation, the T^k matrices R^* are aggregated to get the matrix $\Re$ of economic-financial knowledge.

$$\Re = \begin{array}{c|cccccc|} & c_1 & c_2 & -- & c_j & - & c_p \\ \hline s_1 & \check{r}_{11} & \check{r}_{12} & -- & \check{r}_{1j} & - & \check{r}_{1p} \\ s_2 & \check{r}_{21} & \check{r}_{22} & -- & \check{r}_{2j} & - & \check{r}_{2p} \\ -- & --- & --- & -- & -- & & --- \\ s_i & \check{r}_{i1} & \check{r}_{i2} & -- & \check{r}_{ij} & - & \check{r}_{ip} \\ -- & --- & --- & -- & -- & - & --- \\ s_n & \check{r}_{n1} & \check{r}_{n2} & -- & \check{r}_{nj} & - & \check{r}_{np} \\ \hline \end{array} \qquad (13)$$

These $\check{r}_{ij}$ are consistent and significant, and explain the true relationship between causes and symptoms.

3 APPLICATION OF $\Re$ TO PREDICT DISEASES

The economic-financial knowledge matrix $\Re$ can be used to make economic and financial predictions. That is, given the symptoms of a firm (stated as diverse ratios), $\Re$ can be used to predict the incidence level of each cause c_j defined in the model. Therefore, this methodology allows determining and examining the possible "diseases" that a firm may be suffered. This is proposed through the max-min operation between the membership matrix of symptoms ($Q´$) and the aggregate matrix of economic-financial knowledge ($\Re$) according to (14).

$$P´ = Q´\ \alpha\ \Re = [p´_{hj}] \qquad (14)$$

Being ,

$p´_{hj} = max\ (min\ (q_{hi},\ \check{r}_{ij})),$

Where,

$q´_{ih}$ are the coefficients of incidence of the symptom for any period.

This operation able to detect all the diseases (or causes) of firms into the period that $\Re$ is valid to predict [12], that means two or three years after the estimation of $\Re$.

The estimation of the causes facilitates the early diagnosis of the firm to correct its situation. The degree of adjustment of the diseases is proposed to be tested through a goodness index that compares P with P´, that is $\|P= P'\| = \|P\subseteq P'\| \wedge \|P'\subseteq P\|$ with is represented by (12).

$$[P \;\; =P'] = 1\text{-}1/m \sum_{p\in P}^{m} |p_{hm} \text{-} p'_{hm}| \qquad (15)$$

Although, according to the quantity of causes identified in the model, this could be a difficult task because of the amount of information involved.

4. THE APPLICATION OF OWA

Therefore, in monitoring terms it is useful to concentrate the information in an aggregate number of key areas that represent the disaggregate firm's diseases. The OWA operators [13] are introduced to synthetizes causes and easily detect possible diseases in firms. Once a warning indicator in some area of the firm is detected, it is possible to disaggregate this key area into each of the causes or critical factors that generate problems, in order to evaluate and correct the situation. There are defined a set of key areas, e.g., $W = \{W_z\}$, where $z = 1,2,...,s$; the OWA operators are applied to $\Re$ to calculate the possible diseases of firms.

Mean operator: The $\Re^{Mean}$ matrix is obtained by applying the arithmetic mean to the causes within each monitoring area according to (16).

$$\Re^{Mean} = \sum_{w=1}^{z} \check{r}_{ij} / z \qquad (16)$$

Maximum operator: $\Re$ with maximum membership values of each area ($\Re^{Max}$) are calculated through the maximum selection of $\check{r}_{ij}$ incidence levels within each group of causes.

$$\Re^{Max} = max\,((\check{r}_{ij})_1, (\check{r}_{ij})_2, \ldots, (\check{r}_{ij})_z) \qquad (17)$$

Minimum operator: In this case, $\Re^{Min}$ selects the minimum membership values of each area.

$$\Re^{Min} = min\,((\check{r}_{ij})_1, (\check{r}_{ij})_2, \ldots, (\check{r}_{ij})_z) \qquad (18)$$

This grouping allows the prediction of diseases concentrated in key areas to reduce the information involved. The OWA operators are used to reduce the diseases in key areas. Thus, the membership matrices of causes (or diseases) are obtained in three levels of incidence (minimum (P'^{Min}), maximum (P'^{Max}) and average (P'^{Mean})) (Eq. 19 to Eq. 21), finding the minimum ratio of the operation for each incidence value, depending on the minimum, the maximum or the average..

$$p'^{Min}_{iz} = \Lambda\, [(q_{ih} \circ \check{r}^{min}_{h1}), (q_{ih} \circ \check{r}^{min}_{h2}), ., (q_{ih} \circ \check{r}^{min}_{hz}) \qquad (19)$$

$$p'^{Max}_{iz} = \Lambda\, [(q_{ih} \circ \check{r}^{max}_{h1}), (q_{ih} \circ \check{r}^{max}_{h2}), ., (q_{ih} \circ \check{r}^{max}_{hz}) \qquad (20)$$

$$p'^{Mean}_{iz} = \Lambda\, [(q_{ih} \circ \check{r}^{mean}_{h1}), (q_{ih} \circ \check{r}^{mean}_{h2}), \ldots, (q_{ih} \circ \check{r}^{mean}_{hz}) \qquad (21)$$

The option of monitoring through key areas (consistent with the disaggregated estimation of the model) allows a continuous and comprehensive tracking of the business areas using the fuzzy logic advantages.

4.1. The goodness index

A goodness index is introduced to enrich the diagnostic fuzzy model. This index also is useful to evaluate which the OWA aggregate method is more efficient. The index is based on the Hamming distance, which is adapted to check the functionality of the model and the estimations results. That is, if the estimated causes by the model represent the true situation of the firm.

The index represents the comparison between the original set of causes (P) aggregated into minimum ($P^{Min} = Min\,(p_{hw})$), maximum ($P^{Max} = Max\,(p_{hw})$) and average ($P^{Mean} = (1/z)\,\Sigma\,(p_{hw})$) incidence levels and the set of estimated causes (P´). For this, P´ also is aggregated in the same way as the original causes (P).

Therefore, the goodness index between P and P´ is represented by Eq.22.

$$[P \;\; = P'] = 1 - 1/z \sum_{x\in X}^{z} |p_{hw} - p'_{hw}| \qquad (22)$$

The main advantage of using distance measures in decision making is that is possible to compare the alternatives of the problem with some ideal result, and therefore select the rule with the closest result to the optimal choice.

Therefore,

$$[P= P']^{Min} = [1\text{-}\ 1/z\ \Sigma\,(\,|p_{h1} - p'_{h1}| + |p_{h2} - p'_{h2}| + \ldots + |p_{hw} - p'_{hw}|)]$$

$$[P= P']^{Max} = [1\text{-}\ 1/z\ \Sigma\,(\,|p_{h1} - p'_{h1}| + |p_{h2} - p'_{h2}| + \ldots + |p_{hw} - p'_{hw}|)]$$

$[P= P']^{Mean} = [1- 1/z\ \Sigma(|p_{h1} - p'_{h1}| + |p_{h2} - p'_{h2}| +....+ |p_{hw} - p'_{hw}|)]$

Where,
$P*^{Min}$ selects the minimum degree of incidence within the group of causes for each company;
$P*^{Max}$ chooses the maximum degree of incidence within the group of causes for each company; and,
$P*^{Mean}$ shows the average of the causes within the group or key area of monitoring.

This test is useful to identify the best mechanism for aggregating causes using OWA operators and to estimate the degree of adjustment of the predictions to diseases that are present in the companies. In other words, it is helpful to prove this model's capacity (or any other) to predict insolvency situations and evaluate the three alternatives of synthesizing causes proposed above.

5. ENTROPY AS A MEASURE OF THE CAUSES´ VALUATION

Entropy is a measure of the uncertainty or information contained in a data source, based on considering the average amount of information contained in the symbols used, where symbols with lower probability of occurrence provide more information. In the case where all symbols have the same probability of occurrence (flat probability distribution), then the entropy will be maximum. Unlike other areas of knowledge, in this case, symbols that appear less frequently provide more information. In terms of information theory, in a text, a word that appears only once may be the most important for understanding its meaning.

Entropy is also known as Shannon entropy [14], it is widely used in the field of information theory, both in data coding, cryptography, and communication of information; as well as in fields such as physics and biology to describe the complexity of systems and the information contained in them. It is also used in thermodynamics, statistical mechanics, and entropy-based security. It is mathematically defined as the sum of the probability of each symbol in a set of data multiplied by the negative logarithm of that probability. A greater Shannon entropy suggest a greater degree of uncertainty or amount of information contained in the data.

The traditional entropy measure was developed by [14], but although there have been different complementary measures [15,16,17], it can be affirmed that the most widely used one is Shannon's measure (Eq. 23)

$$H = \sum_{i=1}^{k} P_i . log_2(P_i) \qquad (23)$$

Where,
P_i is the probability of finding the element i in the dataset.

Therefore, if the probability distribution is flat (for example, rolling a die), the P_i values are equal, and therefore the entropy is maximum. That is, the outcome of the experiment is uncertain for each possible extraction of the elements. For example, if we roll a die, the entropy of that experiment according to H is 3.32. However, if for some reason, two faces of that die have triple the probability of occurring $P(0.3; 0.1; 0.1; 0.1; 0.1; 0.3)$, so H is now 2.37. The appearance of differential elements reduces entropy.

In the field of fuzzy mathematics, there has also been development of fuzzy entropies. Fuzzy entropy developed by [18] is presented en Eq. 24.

$$Df = \mathrm{K}. \sum_{h=1}^{k} S(f(x_h) \qquad (24)$$

Where,

Df is the fuzzy entropy, K is a constant, S is the Shannon entropy function, and $f(x_h)$ represents the membership function of each element h of x. [18] prove that Df under the max-min t-norm, satisfies the conditions of a membership function, with some particular limitations.

The calculation of entropy according to the fuzzy definition has different peculiarities. While for Shannon entropy, the maximum entropy for a set of n equal elements is $H = -n\frac{1}{n}\ log2(P_i)$, with $P_i = 1/n$. In the case of fuzzy entropy, this condition is not fulfilled. In H, the summation of $1/n$, in n is equal to 1 (and can never exceed the unit). However, in the case of Df, the sum of the elements of the membership function $f(x_h)$ can exceed the unity, obviously depending on the number of elements considered.

For a set of n=5, the hypothetical value of Df was estimated, assuming that the membership values of Df range between 0.9 and 0.1. The results are presented in table 1.

Membership value	Df
0.9	1.62541487
0.8	2.50201212
0.7	3.05432151
0.6	3.36505834
0.5	3.4657359
0.4	3.36505834
0.3	3.05432151
0.2	2.50201212
0.1	1.62541487

Table 1: The membership value and the fuzzy entropy.

The fuzzy entropy value reaches its maximum level at a membership value of 0.5, which seems very reasonable since a value of 0.5 implies an indeterminacy in the valuation, and therefore, it is the least useful value.

In the case of firms´ diagnosis, fuzzy entropy can be used to analyze expert opinions about the presence of a cause in a particular firm. Let us assume that to construct the Q matrix of the causes, a valuation is carried out with a number F of experts´ opinion about the presence of a cause in a set of m firms. In this case, each element of the $P = [p_{hj}]$ is calculated considering the assessment of the presence of a cause P in each firm doing by each expert in the set F.

$$p_{hj}= \begin{array}{c|cccccc|} & f_1 & f_2 & --- & f_j & --- & f_F \\ E_1 & p_{11} & p_{12} & --- & p_{1j} & --- & p_{1F} \\ E_2 & p_{21} & p_{22} & --- & p_{2j} & --- & p_{2F} \\ --- & --- & --- & --- & --- & --- & --- \\ E_h & p_{h1} & p_{h2} & --- & p_{hj} & --- & p_{hF} \\ --- & --- & --- & --- & --- & --- & --- \\ E_m & p_{m1} & p_{m2} & --- & p_{mj} & --- & p_{mF} \end{array} \quad (25)$$

In this case, the expert assessments are located in each column, and each row represents the assessments of the F experts for the presence of the cause p_{hj} in each firm h.

The use of the Df function can be applied in different ways. Firstly, the presence of the highest level of entropy can be evaluated when the assessments of all experts are close to 0.5. In this case, it would be necessary to consider the treatment given to the cause since it has the maximum level of uncertainty regarding its presence or absence. Another alternative, it is to analyze all the assessments that an expert suggests about each firm. In this case, a certain trend of the expert towards extremes can be analyzed when the entropy value reaches the minimum possible values.

6 CONCLUSIONS

This paper summarizes and complete the fuzzy model of economic and financial diagnosis developed by [3-9]. In this contribution is presented the latest version of the model with all the extensions considered. Lastly, it is proposed the use of entropy measurements of expert opinions, based on a Fuzzy Entropy indicator provided by [18]. The estimation of Fuzzy Entropy allows the visualization of certain problems in the experts' valuation levels, which can cause higher levels of relative uncertainty in the analysis of causes, mainly when the valuation of the cause approaches 0.5.

It should be noted that cause valuation levels close to 0.5 do not provide much utility from the perspective of identifying fuzzy relationships between symptoms and causes. In this case, it may be important to redefine the valuation strategy.

All the upgrades proposed are defined and incorporated to complete this theory, especially in the treatment of causes, because there has been a significant amount of research focused on the evaluation of symptoms and their relationship to causes, the evaluation of causes themselves has received relatively less attention in the literature. Therefore, the accurate identification and evaluation of causes is essential for effective decision-making in the diagnosis and treatment of economic and financial issues in firms. Like in medical diagnosis of a patient, the business diagnosis is done through the relationship between causes and symptoms and the capacity of the expert (or the model) to reduce the significant information for the analysis. This helps in the diagnosis of the disease or in monitoring the treatment to avoid economic and financial distress.

Finally, and although the results are still incipient, it must be analyzed more deeply how to correctly apply the new concept. One possibility is that entropy will be used to evaluate different expert opinions regarding the existence of a disease. However, it is a new field to continue investigating.

REFERENCES

[1] Beaver, W. H. (1966). Financial ratios as predictors of failure. Journal of accounting research, 71-111.

[2] Altman, E. I. (1968). Financial ratios, discriminant analysis and the prediction of corporate bankruptcy. *The journal of finance*, *23*(4), 589-609.

[3] Terceño, A., Vigier, H., & Scherger, V. (2018). Prediction of business failure with fuzzy models. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, *26*(Suppl. 1), 21-38.

[4] Vigier, H. P., & Terceño, A. (2008). A model for the prediction of “diseases” of firms by means of fuzzy relations. *Fuzzy Sets and Systems*, *159*(17), 2299-2316.

[5] Scherger, V., Vigier, H. P., & Barberà-Mariné, M. G. (2014). Finding business failure reasons through a fuzzy model of diagnosis. *Fuzzy economic review*, *19*(1), 45.

[6] Scherger, V., Terceño, A., Vigier, H., & Gloria Barbera-Marine, M. (2015). Detection and Assessment of Causes in Business Diagnosis Economic Computation & Economic Cybernetics Studies & Research, 49(4)..

[7] Scherger, V., Terceño, A., & Vigier, H. P. (2016). Economic-financial diagnosis and prediction of SMEs: An application to the contructin sector. *The International Journal of Management Science and Information Technology (IJMSIT)*, (21), 46-55.

[8] Vigier, H. P., Scherger, V., & Terceño, A. (2017). An application of OWA operators in fuzzy business diagnosis. *Applied Soft Computing*, *54*, 440-448.

[9] Scherger, V., Terceño, A., & Vigier, H. (2017). The OWA distance operator and its application in business failure. *Kybernetes*, *46*(1), 114-130.

[10] Sanchez, E. (1984). Solution of fuzzy equations with extended operations. *Fuzzy sets and Systems*, *12*(3), 237-248.

[11] Zimmermann, H. J. (1987). *Fuzzy sets, decision making, and expert systems* (Vol. 10). Springer Science & Business Media.

[12] Vigier, H. P., & Terceño, A. (2012). Analysis of the inconsistency problem in the model for predicting" diseases" of firms. *Fuzzy Economic Review*, *17*(1), 73.

[13] Yager, R. R. (1993). Families of OWA operators. *Fuzzy sets and systems*, *59*(2), 125-148.

[14] Shannon, C. E. (1948). A mathematical theory of communication. The Bell system technical journal, 27(3), 379-423..

[15] Rényi, A. (1961, January). On measures of entropy and information. In Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability, (Vol. 4, pp. 547-562). University of California Press..

[16] Hartley, R. V. (1928). Transmission of information 1. *Bell System technical journal*, *7*(3), 535-563.

[17] C. Tsallis, Possible generalization of Boltzmann-Tsallis, C. (1988). Possible generalization of Boltzmann-Gibbs statistics. *Journal of statistical physics*, *52*, 479-487.

[18] De Luca, A., & Termini, S. (1972). Algebraic properties of fuzzy sets. *Journal of mathematical analysis and applications*, *40*(2), 373-386.

ANNEX

List of economic and financial ratios (symptoms)

Profitability	Return on assets (EBIT/ Asset) Return on equity (Net profit/ Net equity) Profit margin (Net profit/ Sales) Net profit/ Asset Investment/ Net profit
Productivity	Sales/ number of employees Staff costs/ Sales Staff costs/ Non- current asset
Liquidity	Liquidity (Current asset/ Current liabilities Current asset- stocks/ Current liabilities Cash/ Current liabilities
Leverage, solvency, financial structure and debt coverage	Total debt/ Liabilities Total debt/ Working capital Total debt / Net equity Solvency (Net equity/ Asset) Liabilities/ Net equity Long term debt (Non- current liabilities/ Net equity) Short term debt (Current liabilities/ Net equity) Noncurrent asset/ Asset Working capital/ Sales Current asset- Current liabilities/ Asset Shareholder`s remuneration (Dividends (1-taxes)/ Net equity) Financial costs/ Liabilities Dividends/ Net equity Coverage of interests (EBT/ Interests) EBIT/ Interests
Economic structure	Current Asset/ Non-current asset Quick (Current asset + month sales/(Cost of sales- amortization + adminis-
Activity, turnover and efficiency	Asset turnover (Sales/ Asset) Capital turnover (Sales / Net equity) Stocks/ Sales Cost of sales/ Stocks Leverage (Asset/ Net equity * EBT/EBIT) Current asset/ Sales Sales/ Non-current asset Account receivable/ Sales Account receivable/ Stocks Operating income/ operating costs
Self- financing	Retained earnings/ Asset Retained earnings/ Net equity Lag without credit ((Current asset – Stocks – Current liabilities) / (Operating